Research on Missile Control System with a Kind of Fractional Order Differential Control Method

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Abstract. Based on the common PID controller design method, a kind of fractional order controller is designed for a missile system with linear pitch channel model. And detailed simulation is done to testify the proposed method and it shows that the fractional order derivative can also provide the enough damping ratio. So although the fractional order derivative is more complex than the common PID, and it can make the system more stable.

Introduction

The analysis and design of control systems need simple and accurate mathematical models. However, for practical systems with more complex characteristics, it is often difficult to give consideration to both requirements by using traditional integer order methods. Fractional calculus has the following three advantages [1-4]: 1) Fractional derivatives are globally correlated and more suitable for describing systems with historical dependence process, while integer derivatives are only local, so they can not better reflect the historical dependence process of system development; 2) Fractional derivatives have the characteristics of both finite and infinite dimensional systems, which are bridges between the two systems and can overcome traditional integers. The theory of order calculus model can not describe the difficulty of distributed parameter system [5-9]. 3) Traditional integer-order controller is a special case of fractional-order controller. The parameters of fractional-order controller have more degrees of freedom, which can achieve the performance that traditional controller can not achieve (better robustness and transient performance), such as CRONE controller and P11D controller [10-16], which have been obtained in engineering practice.

With the development of weapons and equipment, missile weapons play an increasingly important role in the military equipment of all countries in the world. At present, most of the traditional missile weapons adopt PID controller [17-22]. Considering that the PID control is only a special case of fractional order PID control, this paper attempts to introduce fractional order PID control algorithm into the missile weapon and try to improve the performance of the missile control system through the design of fractional order PID controller.

The Definition of Fractional Order Differential and System Model

Mathematicians have defined fractional calculus from different angles. At present, there are three definitions of fractional calculus including G. L definition, R. L definition and Caputo definition.

The GL definition of fractional differential is defined as

$$\frac{d^q x}{dt^q} = f(t, x) \tag{1}$$

Where q is the order of fractional differential system.

The nonlinear pitch channel model of a kind of missile system can be described as

$$\begin{cases} \dot{\alpha} = \omega_z - \dfrac{qS}{mv} C_y(M_m, \rho, \alpha, \omega_z, \delta_z) \\ \dot{\omega}_z = \dfrac{qSL}{J_z} C_m(M_m, \rho, \alpha, \omega_z, \delta_z) \\ A_{yB} = \dfrac{qS}{m} C_y(M_m, \rho, \alpha, \omega_z, \delta_z) = n_{yB} g \end{cases} \tag{2}$$

And if the attack angle is very small, then its linear model can be written as

$$\begin{cases} \dot{\alpha} = \omega_z - a_{34}\alpha - a_{35}\delta_z = \omega_z - \dfrac{g}{v} n_y \\ \dot{\omega}_z = a_{24}\alpha + a_{22}\omega_z + a_{25}\delta_z \\ n_y = \dfrac{v}{g} a_{34}\alpha + \dfrac{v}{g} a_{35}\delta_z \end{cases} \tag{3}$$

And the control objective is to design a fractional order controller to make the missile attack angle to track the desired value.

Calculation of Fractional Derivative of Sine Function

According to the definition of fractional order derivative, it has

$$D_t^{\alpha} f(t) = \frac{1}{h^{\alpha}} \sum_{j=0}^{\frac{t-t_0}{h}} (-1)^j C_j^{\alpha} f(t - jh) \tag{4}$$

We take a sine function as an example to solve the fractional order derivative and write a program as follows

```
for i=0:tf/h
    sf=0;    t=i*h;
    for j=0:i
        Calfaj=gamma(alfa+1)/(gamma(j+1)*gamma(alfa-j+1));
        tjh=t-j*h;
        ftjh=sin(tjh);
        sf=sf+(-1)^j*Calfaj*ftjh;
    end
    dalfaft=1/(h^(alfa))*sf;
    tp(i+1)=t;
    dalfaftp(i+1)=dalfaft;
    d=d+1;
    if d==10
        waitbar(i/(tf/h));
        d=1;
    end
end
close(w)
figure(1)
plot(tp,dalfaftp)
```

Fractional Order Controller Design for Missile System

For above missile system, we design a common PID controller as following

$$u = k_p e + k_i \int edt + k_d \dot{e} \tag{5}$$

Where $e = \alpha - \alpha^d$. And it can be improved to be a fractional controller as

$$u = k_p e + k_i \int edt + k_d \dot{e} + k_\alpha e^{(\gamma)} \tag{6}$$

Where $0 < \gamma < 1$, and

$$\begin{cases} e^{(\gamma)} = D_t^\alpha f(t) \\ f(t) = e(t) \\ D_t^\alpha f(t) = \dfrac{1}{h^\alpha} \displaystyle\sum_{j=0}^{(t-t_0)/h} (-1)^j C_j^\alpha f(t - jh) \end{cases} \tag{7}$$

So the error function is e(n)and in the program we set e(t-jh)equal to e(n-j) because the array is begin from No one but not from zero. So the data e(n-j) is in fact storage in e(n-j+1), which should be very careful.

Simulation Result

Finally, the simulation result can be shown as below figure 1, and the response time for the missile system is 0.1 s. So the quickness of the controller is very good and we found that if the fractional order is set to be close to 1, the fractional order derivative can provide enough damping for the system just like the integer order derivative.

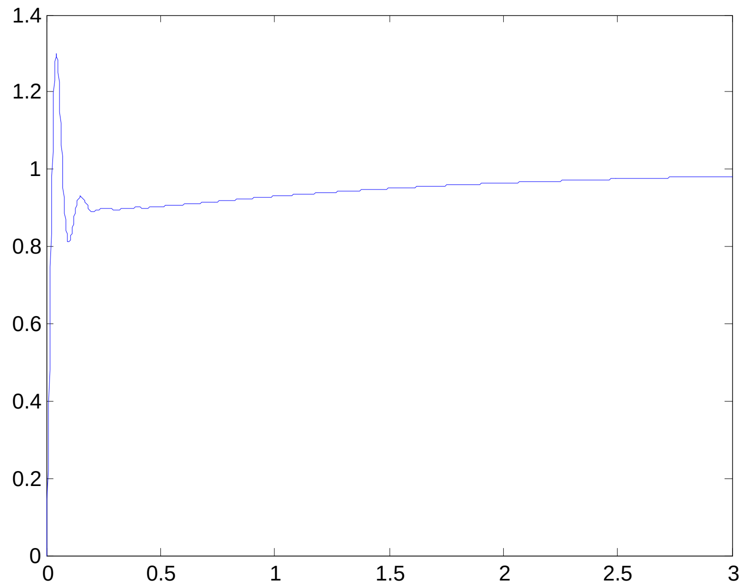

Figure 1 Output of missile system (attack angle)

Conclusion

In this paper, by introducing fractional differential of error signal, a fractional order PID controller is designed for a class of missile pitch channel control system. The controller can be applied not only to simplified linear model, but also to real non-linear missiles. The simulation results show that the fractional differential has better damping characteristics when the order is close to 1.

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